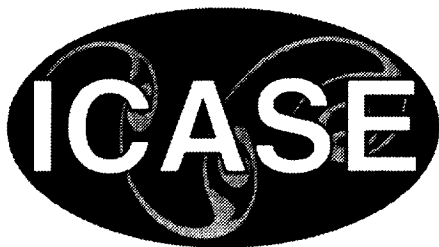


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DOUBLE RESONANCES AND SPECTRAL SCALING IN THE WEAK TURBULENCE THEORY OF ROTATING AND STRATIFIED TURBULENCE

ROBERT RUBINSTEIN*

Abstract. In rotating turbulence, stably stratified turbulence, and in rotating stratified turbulence, heuristic arguments concerning the turbulent time scale suggest that the inertial range energy spectrum scales as k^{-2} . From the viewpoint of weak turbulence theory, there are three possibilities which might invalidate these arguments: four-wave interactions could dominate three-wave interactions leading to a modified inertial range energy balance, double resonances could alter the time scale, and the energy flux integral might not converge. It is shown that although double resonances exist in all of these problems, they do not influence overall energy transfer. However, the resonance conditions cause the flux integral for rotating turbulence to diverge logarithmically when evaluated for a k^{-2} energy spectrum; therefore, this spectrum requires logarithmic corrections. Finally, the role of four-wave interactions is briefly discussed.

Key words. rotating turbulence, stratified turbulence, weak turbulence

Subject classification. Physical Sciences

1. Introduction. Weak turbulence theory [1, 2, 3] provides a closure for problems of interacting dispersive waves. As developed by Zakharov and collaborators [3], it provides a means to calculate Kolmogorov-like spectra carrying constant fluxes of invariant quantities, to decide whether these spectra are independent of the large-scale excitation and the dissipative mechanism (the problem of *locality*), to decide whether energy, or other invariants, are carried from large to small scales or from small to large scales (*direct* or *inverse* cascades), and to assess the stability of these spectra to various perturbations. A two-point closure closely related to weak turbulence theory has been applied by Cambon and co-investigators to the numerical simulation of rotating [4] and of stratified [5] turbulence.

An important ingredient in this theory is the *resonance operator*

$$(1.1) \quad \mathcal{R}(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \int_{-\infty}^{\infty} d\tau e^{i\tau h(\mathbf{k}, \mathbf{p}, \mathbf{q})} \delta(\mathbf{k} - \mathbf{p} - \mathbf{q})$$

where h is defined in terms of the dispersion relation $\omega = \omega(\mathbf{k})$ of the waves by

$$(1.2) \quad h(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \omega(\mathbf{k}) \pm \omega(\mathbf{p}) \pm \omega(\mathbf{q})$$

Eq. (1.1) has been written for a quadratically nonlinear theory, in which the lowest order perturbation theory [2, 3] leads to three-wave interactions. Note that each choice of signs in Eq. (1.2) leads to a different operator.

The resonance operator defined by Eq. (1.1) depends on the geometric properties of the *resonance surface* $h = 0$. The most important question about the resonance surface is whether it contains any (real) points at all. If $h(\mathbf{k}, \mathbf{p}, \mathbf{q})$ never vanishes when $\mathbf{k} = \mathbf{p} + \mathbf{q}$, then the integrand in Eq. (1.1) oscillates strongly and the resonance operator vanishes. If all three-wave resonance operators vanish, it is necessary to proceed to the next order in perturbation theory, which generates a theory with a nonlinearity of higher order than

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the original equations of motion; for quadratically nonlinear problems, the result is a theory with a cubic nonlinearity. In such a theory of four-wave interactions, the resonance surface is never empty [3].

Assuming that the resonance surface is nonempty, the next question is whether it is nonsingular. If it is, then elementary partition of unity arguments [6] show that

$$(1.3) \quad \int_{-\infty}^{\infty} d\tau e^{i\tau h(\mathbf{k}, \mathbf{p}, \mathbf{q})} = \delta(h)$$

Consequently, if the dispersion relation is homogeneous of degree α ,

$$(1.4) \quad \omega(\lambda \mathbf{k}) = \lambda^\alpha \omega(\mathbf{k})$$

then

$$(1.5) \quad \mathcal{R}(\lambda \mathbf{k}, \lambda \mathbf{p}, \lambda \mathbf{q}) = \lambda^{-d-\alpha} \mathcal{R}(\mathbf{k}, \mathbf{p}, \mathbf{q})$$

where d is the dimension of space. Eq. (1.4) states that the turbulent time scale is of order $k^{-\alpha}$. For problems with three-wave interactions, including problems with hydrodynamic nonlinearity, a simple scaling argument given by Orszag [7] suggests that when the energy flux is constant, the spectrum scales as

$$(1.6) \quad E(k) \sim k^{-2+\alpha/2}$$

A complication which these arguments ignore is the possibility that all three-wave resonance operators vanish. As noted earlier, turbulent energy transfer is then dominated by four-wave interactions and the inertial range energy balance which leads to Eq. (1.6) must be modified. For later reference, we note the energy spectrum corresponding to four-wave interactions

$$(1.7) \quad E(k) \sim k^{-7/3+\alpha/3}$$

Another complication which is ignored by the heuristic arguments is the existence of *double resonances*, which arise whenever the resonance surface is singular. The arguments leading to Eq. (1.3) are no longer valid, and the theory leads to a more complex resonance operator and possibly to a different time scale. The classic case of double resonance is nonlinear sound waves [8, 9]. In two space dimensions, the double resonances change the time scale of the problem completely, and the scaling suggested by Eq. (1.6) does not apply [3]. This problem will be discussed in more detail in the next section.

The dimensional arguments leading to Eq. (1.6) also assume that no time scale except the linear dispersive time scale is relevant, and that no length scale except the wavelength $\sim k^{-1}$ is relevant; constancy of the energy flux then suffices to determine the spectral scaling. In particular, neither the production nor the dissipation mechanism influences the spectral scaling. This *locality* condition noted earlier can be expressed analytically as the convergence of an integral representing the energy flux when this flux integral is evaluated for an infinite inertial range given by Eq. (1.6). Divergence at large or small scales indicates the relevance of the production or dissipation mechanism respectively, and alters the spectral scaling. Locality depends on the geometry of the resonance surface, since the flux integral might diverge along singular curves or points of the surface.

This paper considers three related problems of weak turbulence theory: rotating turbulence, stratified turbulence, and the coupled problem of rotating, stratified turbulence. In all three problems, the dispersion relation is homogeneous of degree zero:

$$(1.8) \quad \omega(\lambda \mathbf{k}) = \omega(\mathbf{k})$$

In the notation of Eq. (1.4), $\alpha = 0$; consequently Eq. (1.6) predicts

$$(1.9) \quad E(k) \sim k^{-2}$$

The k^{-2} spectrum has been already been proposed for rotating turbulence by Zhou [10] and for stratified turbulence by Herring [11] using arguments similar to those of Orszag [7] noted earlier. Although all three problems contain a preferred direction, and must exhibit spectral anisotropy, we consider in Eq. (1.9) the energy averaged over shells in wavevector space.

The question which we pose is whether the scaling Eq. (1.9) actually is the prediction of weak turbulence theory for these problems. In order that this be so, it is necessary to

1. evaluate the effect of any singularities of the resonance surface
2. demonstrate the convergence of the flux integral.
3. demonstrate the applicability of three-wave theory

It will be shown that although double resonances exist in all of these problems, they do not influence energy transfer; consequently, the double resonances do not alter the energy spectrum. But evaluation of the energy flux integral for rotating turbulence shows that the k^{-2} spectrum is only marginally local, and that logarithmic corrections analogous to those derived by Kraichnan [12] for the enstrophy range in two-dimensional turbulence are necessary. Accordingly, instead of Eq. (1.9),

$$(1.10) \quad E(k) \sim k^{-2} [\log(kL_0)]^{-1/2}$$

where L_0 is the integral scale. Finally, the role of four-wave interactions in these problems is briefly addressed. These problems admit three-wave interactions, except for one special case of coupled rotating, buoyant turbulence.

Since the k^{-2} spectrum also occurs in Burgers turbulence, a connection is sometimes made between the occurrence of this spectrum and the presence of wave breaking [13]. The physical origin of the k^{-2} spectrum in weak turbulence theory is certainly unrelated to wave breaking, which is an effect of strong nonlinearity. In this connection, it is interesting that Kraichnan's derivation [14] of the spectrum of Burgers turbulence does not invoke wave-breaking, although it is also a strong turbulence result.

The analysis of this paper will be limited to wave interactions alone: it will only consider the turbulence of waves satisfying the dispersion relation appropriate to each problem. This is certainly a limitation in the cases of stratified turbulence and the coupled problem, in which interactions with non-oscillatory geostrophic modes [13] dominate the dynamics [15]. Since there is no three-wave mechanism which generates these modes from wave interactions [13], this analysis appears to be self-consistent. However, the question is very much open whether four-wave interactions cannot generate geostrophic modes [16], an interaction which would have considerable importance in geophysical applications.

2. Double resonances in weak turbulence theory. The classic case of double resonances in weak turbulence theory occurs for nonlinear sound waves [8, 9], where

$$(2.1) \quad \omega(\mathbf{k}) = c |\mathbf{k}|$$

Double resonances occur if

$$(2.2) \quad \begin{aligned} \frac{p_x}{p} \pm \frac{q_x}{q} &= 0 \\ \frac{p_y}{p} \pm \frac{q_y}{q} &= 0 \\ \frac{p_z}{p} \pm \frac{q_z}{q} &= 0 \end{aligned}$$

These conditions are equivalent to

$$(2.3) \quad \pm \frac{p}{q} = \frac{p_x}{q_x} = \frac{p_y}{q_y} = \frac{p_z}{q_z}$$

and can be satisfied for any \mathbf{k} if \mathbf{p}, \mathbf{q} and \mathbf{k} are collinear.

In three dimensions, the double resonance induces a logarithmic time scale which requires a re-ordering of the perturbation theory [8] on which weak turbulence theory is based, but it does not alter the scaling relation Eq. (1.5) or the spectral scaling, $E(k) \sim k^{-3/2}$ which Eq. (1.6) suggests since $\omega \sim k^1$.

In two dimensions, however, the situation is quite different: the resonance operator proves to scale with time as $t^{1/2}$, and the theory must be reformulated completely [9]. Thus, if the $t^{1/2}$ time dependence is absorbed into the definition of a new slow time scale, the resonance operator would scale as $\mathcal{R} \sim (k/c)^{1/2}$ instead of as $\mathcal{R} \sim k/c$, leading to $E(k) \sim k^{-7/4}$.

The problem of nonlinear sound waves is exceptional in several respects [3], and the correct spectral scaling is now considered to depend on higher order terms in the dispersion relation [17]. Nevertheless, this example illustrates how double resonances can modify the turbulent time scales and consequently alter the predictions of weak turbulence theory. We next determine whether singularities exist in the resonance surface for the three problems of this paper.

3. Resonance surface singularities. The three cases of rotating, stratified, and rotating stratified turbulence will be considered in turn. Although rotation and stratification are formally special cases of the coupled problem, they prove to have special features requiring separate analysis.

3.1. Rotating turbulence. The dispersion relation of inertial waves in rotating turbulence is

$$(3.1) \quad \omega(\mathbf{k}) = \pm 2\Omega \frac{k_z}{k}$$

where Ω is the rotation rate. The resonance surface is therefore

$$(3.2) \quad h(\mathbf{k}, \mathbf{p}, \mathbf{q}) = \frac{k_z}{k} \pm \frac{p_z}{p} \pm \frac{q_z}{q} = 0$$

Note that the resonance surface is nonempty and three-wave interactions are possible in this problem.

We must consider the possible singularities of the surface $h = 0$ when $\mathbf{k}, \mathbf{p}, \mathbf{q}$ satisfy the triangle condition $\mathbf{k} = \mathbf{p} + \mathbf{q}$. Singularities, or double resonances, occur if $\partial h / \partial \mathbf{p} = 0$, or

$$(3.3) \quad \frac{\partial}{\partial \mathbf{p}} \left(\frac{p_z}{p} \pm \frac{q_z}{q} \right) = 0$$

when $h = 0$. Eq. (3.3) states that

$$(3.4) \quad \begin{aligned} \frac{p_x p_z}{p^3} &= \pm \frac{q_x q_z}{q^3} \\ \frac{p_y p_z}{p^3} &= \pm \frac{q_y q_z}{q^3} \\ \frac{p_x^2 + p_y^2}{p^3} &= \pm \frac{q_x^2 + q_y^2}{q^3} \end{aligned}$$

It is understood that the sign in each equality in Eq. (3.4) is the same, hence only two cases are represented. But the last equality can only be satisfied if both sides are positive; therefore, the negative sign case in Eq. (3.4) does not give any double resonance.

Assume first that no component of \mathbf{p} vanishes. Rearranging the terms,

$$(3.5) \quad \frac{q^3}{p^3} = \frac{q_x q_z}{p_x p_z} = \frac{q_y q_z}{p_y p_z} = \frac{q_x^2 + q_y^2}{p_x^2 + p_y^2}$$

therefore

$$(3.6) \quad \frac{q_x}{p_x} = \frac{q_y}{p_y} = \frac{q^3}{p^3} \frac{p_z}{q_z}$$

and

$$(3.7) \quad \frac{q^3}{p^3} = \frac{q^6}{p^6} \frac{p_z^2}{q_z^2}$$

consequently

$$(3.8) \quad \frac{q_x}{p_x} = \frac{q_y}{p_y} = \frac{q_z}{p_z}$$

The homogeneity of the dispersion relation means that the first set of equalities can only be satisfied if

$$(3.9) \quad \frac{q}{p} = 1$$

Note the role of the homogeneity of this problem $\omega \sim k^0$ in forcing this conclusion: the result Eq. (3.9) does not apply to sound waves, because $\omega \sim k^1$.

We conclude that the conditions for double resonance can only be satisfied if either $\mathbf{q} = \mathbf{p}$ or $\mathbf{q} = -\mathbf{p}$. In the first case in which $\mathbf{k} = 2\mathbf{p}$, the wavevectors are collinear, and the dispersion relation requires $\pm 1 \pm 1 \pm 1 = 0$, which is impossible. The second case in which $\mathbf{k} = 0$ pertains to a degenerate mode which we do not consider. It follows that double resonances can only occur if some component of \mathbf{p} vanishes.

Next, consider these special solutions of Eq. (3.4). There are two obvious solutions which satisfy the resonance conditions. First, if $p_z = q_z = 0$ and $p = q$, the resonance conditions are trivially satisfied, since also $k_z = 0$. Under these conditions, $p = q$ implies that \mathbf{p} lies on the line

$$(3.10) \quad 2\mathbf{p} \cdot \mathbf{k} = k^2, p_z = 0$$

If $k_z = 0$, the resonance conditions require that either $p_z = q_z = 0$, or that $2\mathbf{p} \cdot \mathbf{k} = k^2$. These planes intersect in the line defined by Eq. (3.10), which is therefore a double line of the resonance surface.

A second obvious solution of Eq. (3.4) is $p^\perp = q^\perp = 0$, where $(p^\perp)^2 = p_x^2 + p_y^2$. In this case, the resonance condition cannot be satisfied if \mathbf{k} is a third vertical vector, because, as noted above, the resonance condition and collinearity would imply $\pm 1 \pm 1 \pm 1 = 0$. The only possibility is that $k_z = 0$. But then the triad condition $\mathbf{k} = \mathbf{p} + \mathbf{q}$ forces $\mathbf{k} = 0$. Therefore, we again are led to resonance with a degenerate mode, a case which can be ignored.

The remaining instances of vanishing components do not lead to double resonances not already noted: the elementary case-by-case verification appears in Appendix I. Accordingly, we conclude that the resonance surface has exactly one singularity, the double line defined by Eq. (3.10).

3.2. Waves in stratified turbulence. Internal waves in stratified turbulence satisfy the dispersion relation

$$(3.11) \quad \omega(\mathbf{k}) = \pm N \frac{k^\perp}{k}$$

Three-wave interactions are possible for this problem.

For a double resonance,

$$(3.12) \quad \begin{aligned} \frac{p_x p_z^2}{p^3 p^\perp} &= \pm \frac{q_x q_z^2}{q^3 q^\perp} \\ \frac{p_y p_z^2}{p^3 p^\perp} &= \pm \frac{q_y q_z^2}{q^3 q^\perp} \\ \frac{p_z p^\perp}{p^3} &= \pm \frac{q_z q^\perp}{q^3} \end{aligned}$$

and $h = \omega(\mathbf{p}) \pm \omega(\mathbf{q}) \pm \omega(\mathbf{k}) = 0$. As in Eq. (3.4), only two cases are represented by Eq. (3.12). From the first two equalities in Eq. (3.12),

$$(3.13) \quad \frac{p_x}{q_x} = \frac{p_y}{q_y} = \pm \frac{p^3 p^\perp q_z^2}{q^3 q^\perp p_z^2}$$

Using the third relation of Eq. (3.12),

$$(3.14) \quad \pm \frac{p^3 p^\perp q_z^2}{q^3 q^\perp p_z^2} = \pm \frac{p^3 q_z p^\perp q_z}{p_z q^3 q^\perp p_z} = \left(\frac{p^\perp}{q^\perp}\right)^2 \frac{q_z}{p_z}$$

where only the positive sign applies in the last equality. Since

$$(3.15) \quad \frac{p_x}{q_x} = \frac{p_y}{q_y} = \pm \frac{p^\perp}{q^\perp}$$

the last equality implies also

$$(3.16) \quad \pm \frac{p^3 p^\perp q_z^2}{q^3 q^\perp p_z^2} = \left(\frac{p^\perp}{q^\perp}\right)^2 \frac{q_z}{p_z} = \frac{p_x^2 q_z}{q_z^2 p_z}$$

and finally, as in Eq. (3.8),

$$(3.17) \quad \frac{p_x}{q_x} = \frac{p_y}{q_y} = \frac{p_z}{q_z}$$

Since the homogeneity implies that necessarily $q/p = 1$, we are again led to the two cases considered in the previous section. The dispersion relation again excludes the case of collinearity $\mathbf{p} = \mathbf{q}$ and $\mathbf{k} = 2\mathbf{p}$. If $\mathbf{q} = -\mathbf{p}$, we obtain a degenerate mode $\mathbf{k} = 0$ and this case is again excluded.

Next, consider the solutions of Eq. (3.12) when at least one component of \mathbf{p} vanishes. First, if $p_z = q_z = 0$, then also $k_z = 0$, and the resonance condition cannot be satisfied since it requires $\pm 1 \pm 1 \pm 1 = 0$.

Second, if $p^\perp = q^\perp = 0$, the third equation is satisfied but the first two become indeterminate. But it is clear that three vertical vectors satisfying the triad condition are always resonant. Such vectors define a degenerate line component of the resonance surface; any such component must be singular.

The remaining cases of vanishing components do not lead to anything new, and are analyzed in Appendix II.

3.3. Waves in rotating stratified turbulence. The dispersion relation is

$$(3.18) \quad \begin{aligned} \omega(\mathbf{k})^2 &= 4\Omega^2 \left(\frac{k_z}{k}\right)^2 + N^2 \left(\frac{k^\perp}{k}\right)^2 \\ &= (4\Omega^2 - N^2) \left(\frac{k_z}{k}\right)^2 + N^2 \end{aligned}$$

Three-wave interactions are possible provided $2\Omega \neq N$. When $2\Omega = N$, two waves can only interact with a geostrophic mode [13] and interactions among waves alone must be described by a four-wave theory.

Double resonances occur provided

$$(3.19) \quad \begin{aligned} \frac{1}{\omega_p} \frac{p_x p_z^2}{p^4} &= \pm \frac{1}{\omega_q} \frac{q_x q_z^2}{q^4} \\ \frac{1}{\omega_p} \frac{p_y p_z^2}{p^4} &= \pm \frac{1}{\omega_q} \frac{q_y q_z^2}{q^4} \\ \frac{1}{\omega_p} \frac{p_z (p^\perp)^2}{p^4} &= \pm \frac{1}{\omega_q} \frac{q_z (q^\perp)^2}{q^4} \end{aligned}$$

As in the two previous cases,

$$(3.20) \quad \frac{p_x}{q_x} = \frac{p_y}{q_y} = \pm \frac{\omega_p}{\omega_q} \frac{p^4}{q^4} \frac{q_z^2}{p_z^2}$$

Using the third relation from Eq. (3.19),

$$(3.21) \quad 1 = \pm \frac{\omega_p}{\omega_q} \frac{q_z}{p_z} \left(\frac{q^\perp}{p^\perp} \right)^2 \frac{q^4}{p^4}$$

consequently,

$$(3.22) \quad \frac{p_x}{q_x} = \frac{p_y}{q_y} = \frac{q_z}{p_z} \left(\frac{p^\perp}{q^\perp} \right)^2 = \frac{q_z}{p_z} \frac{p_x^2}{q_x^2}$$

so that the conclusion reached in the two special cases,

$$(3.23) \quad \frac{p_x}{q_x} = \frac{p_y}{q_y} = \frac{p_z}{q_z}$$

again holds. Since the homogeneity of the dispersion relation requires that the factor of proportionality in Eq. (3.23) have magnitude one, we again come to the two cases before, neither of which leads to a double resonance.

The solutions of Eq. (3.19) when at least one component of \mathbf{p} vanishes include first $p_z = q_z = 0$. Then also $k_z = 0$ and the resonance conditions cannot be satisfied since they reduce to $\pm 1 \pm 1 \pm 1 = 0$. The same is true of the second obvious solution $p^\perp = q^\perp = 0$. Thus, neither of the cases which lead to double resonances in the uncoupled problems remains in the coupled problem.

However, the solution $p_x = p_y = 0$, $q_z = 0$ does lead to a double resonance. In this case, the resonance condition reduces to

$$(3.24) \quad 4\Omega^2 p_z^2 + N^2 (q^\perp)^2 = k^2 (2\Omega \pm N)^2$$

and the choice of the negative sign gives the equation

$$(3.25) \quad 0 = k_z^2 (N^2 - 4\Omega N) + (k_x^2 + k_y^2) (4\Omega^2 - 4\Omega N)$$

since $\mathbf{k} = (q_x, q_y, p_z)$. Thus, for every \mathbf{k} satisfying Eq. (3.25), the decomposition $\mathbf{p} = (0, 0, p_z)$, $\mathbf{q} = (k_x, k_y, 0)$ defines a double resonance. Eq. (3.25) is the equation of a cone provided $N > \Omega$ or $N < 4\Omega$; if $\Omega < N < 4\Omega$, then there are no double resonances at all. The second alternative applies to the case of four-wave resonance $N = 2\Omega$ noted earlier. If we consider the general case in which $q_z = 0$ and look near the point $p_x = p_y = 0$ when \mathbf{k} satisfies Eq. (3.25), it is found that the singularity is that of a line component, the same singularity that occurs in stratified turbulence.

The remaining case in which Eq. (3.19) can be satisfied is $p_x = q_x = 0$. As in the previous cases, this reduces to the case of collinearity, which has already been considered.

4. Singularities and the resonance operator. To determine the effect of the multiple resonances, it is necessary to consider the form of the resonance operator near the singularity. For example, in the case of nonlinear sound waves, all resonances are double because the resonance condition forces collinearity of the three wavevectors. Locally, the resonance operator behaves like the generalized function

$$(4.1) \quad [\mathcal{D}, f] = \int dV \int_{-\infty}^{\infty} d\tau e^{i(x^2+y^2)\tau} f(x, y, z)$$

in which the resonance surface has degenerated into the line $x = y = 0$. In Eq. (4.1), $f(x, y, z)$ is an arbitrary test function. Changing to polar coordinates,

$$(4.2) \quad \begin{aligned} [\mathcal{D}, f] &= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dz \int_0^{\infty} r dr \int_0^{2\pi} d\theta e^{ir^2\tau} f(r, \theta, z) \\ &= \int_{-\infty}^{\infty} dz f(0, 0, z) \end{aligned}$$

Consequently, [8] the resonance operator reduces to integration along the line representing the degenerate resonance surface. The three distinct integrals in Ref. 8 correspond to the different arrangements of three collinear vectors: $k > p > q$, $p > k > q$, or $p > q > k$.

This same approach will be applied to analyze the effect of double resonances in rotating and stratified turbulence: the resonance operator will be represented locally by a phase function with the same singularity as the resonance surface. Since the double locus is different in rotating and stratified turbulence, each case must be analyzed separately.

4.1. Rotating turbulence. Unlike the case of nonlinear sound waves, the resonance surface is non-degenerate for rotating turbulence, and it is nonsingular away from the double curve. Consequently, one contribution to the resonance operator is just $\delta(h)$. The problem remains to determine the effect of the double curve on the resonance operator.

Near a double curve, the resonance operator behaves like the generalized function

$$(4.3) \quad [\mathcal{D}, f] = \int dV \int_0^{\infty} d\tau e^{ixy\tau} f(x, y, z)$$

in which $x = y = 0$ is a double line of the surface $xy = 0$. In polar coordinates,

$$(4.4) \quad [\mathcal{D}, f] = \int_0^{\infty} d\tau \int_{-\infty}^{\infty} dz \int_0^{\infty} r dr \int_0^{2\pi} d\theta e^{ir^2 \frac{1}{2} \sin(2\theta)\tau} f(r, \theta, z)$$

To determine the effect of the double curve, it will suffice to ignore the variation along the double curve and evaluate

$$(4.5) \quad [\mathcal{D}^*, f] = \int_0^{\infty} d\tau \int_0^{\infty} r dr \int_0^{2\pi} d\theta e^{ir^2 \frac{1}{2} \sin(2\theta)\tau} f(r, \theta)$$

in the special case $f = \exp(-\frac{1}{2}ar^2)$. If the limit

$$(4.6) \quad \lim_{a \rightarrow \infty} [\mathcal{D}^*, e^{-ar^2}]$$

is zero, the double curve makes no contribution to the resonance operator; if it is not zero, then there is an additional contribution corresponding to integration over the double curve.

Integrating over r ,

$$(4.7) \quad [\mathcal{D}^*, e^{-ar^2}] = \int_0^{2\pi} d\theta \int_0^{\infty} d\tau \frac{1}{a - i\tau \sin(2\theta)}$$

The imaginary part of this integral is logarithmic, but it makes no contribution to the energy balance [2]. The real part of the integral in Eq. (4.7) can be evaluated as

$$(4.8) \quad \text{Re}[\mathcal{D}^*, e^{-ar^2}] = \lim_{T \rightarrow \infty} 8 \int_{-\pi/4}^{\pi/4} \frac{d\theta}{\sin(2\theta)} \arctan\left(\frac{T}{a} \sin(2\theta)\right)$$

The integral can be evaluated approximately by setting

$$(4.9) \quad \arctan\left(\frac{T}{a} \sin(2\theta)\right) \approx \begin{cases} T \sin(2\theta)/a & \text{if } |\sin(2\theta)| \leq \pi a/2T \\ \pi/2 & \text{if } |\sin(2\theta)| \geq \pi a/2T. \end{cases}$$

Substituting Eq. (4.9) in Eq. (4.8),

$$(4.10) \quad [\mathcal{D}^*, e^{-ar^2}] \approx \pi + \lim_{T \rightarrow \infty} 2\pi \log\left(\frac{a\pi}{2T}\right)$$

This approximation defines an upper bound, since the left side of Eq. (4.9) is smaller than the right side.

We conclude that the double curve contribution to the resonance operator is logarithmically divergent in time. Therefore, for rotating turbulence, the real part of the resonance operator is

$$(4.11) \quad \text{Re}\mathcal{R} = \frac{1}{\Omega} \sum_{\pm\pm} \delta(\omega(\mathbf{k}) \pm \omega(\mathbf{p}) \pm \omega(\mathbf{q})) \delta(\mathbf{k} - \mathbf{p} - \mathbf{q}) + T^* \delta(C)$$

where C denotes the double curve Eq. (3.10). The notation T^* is used provisionally to represent the logarithmically divergent time scale.

In terms of the multiple-scale perturbation expansion used in weak turbulence theory, the result in Eq. (4.11) should be understood to mean that there are two slow time scales in rotating turbulence,

$$(4.12) \quad T_1 = \epsilon^2 t, \quad T_2 = \epsilon^2 \int_1^t \log(s) ds$$

where ϵ is the expansion parameter [2]. Unlike the otherwise analogous case of sound waves, this logarithmic time scale appears in the real part of the resonance operator and therefore could influence the inertial range energy balance.

However, there is an important difference between the present case and sound waves: whereas in the problem of nonlinear sound waves, double resonances occur for all possible \mathbf{k} , in rotation, they occur only for horizontal vectors. The weak logarithmic singularity induced by the double resonance will disappear upon integration over all vectors \mathbf{k} in the flux integral. Moreover, the double resonance occurs for interactions which do not transfer any energy into the mode \mathbf{k} [18]. We conclude that the double curve makes no contribution to the energy flux.

4.2. Waves in stratified turbulence and rotating stratified turbulence. In both problems, the Taylor series of the resonance surface near the singularity begins with the quadratic term $p_x^2 + p_y^2$ which is satisfied by the line $p_x = p_y = 0$. This is also the singularity found in the case of nonlinear sound waves. The local behavior of the resonance operator is therefore given by Eq. (4.1) and the conclusions of Newell and Aucoin [8] apply: the double resonance causes a logarithmic time scale, but does not modify the spectral scaling.

But, as in the case of rotation, the logarithmic time-scale arises only for a subset of vectors \mathbf{k} satisfying at least one equation: vertical vectors in the case of stratified turbulence, and the condition Eq. (3.25) in the coupled problem. In forming the flux integral, the logarithmic contributions to the time-scale will be suppressed by integration over all vectors \mathbf{k} .

We conclude that the double resonances are irrelevant to energy transfer and spectral scaling in all three problems.

5. Convergence of the flux integral. In rotating turbulence, it is convenient to express the flux integral in terms of the *Craya-Herring* basis

$$(5.1) \quad \begin{aligned} \mathbf{e}^{(1)}(\mathbf{k}) &= \mathbf{k} \times \boldsymbol{\Omega} / |\mathbf{k} \times \boldsymbol{\Omega}| \\ \mathbf{e}^{(2)}(\mathbf{k}) &= \mathbf{k} \times (\mathbf{k} \times \boldsymbol{\Omega}) / |\mathbf{k} \times (\mathbf{k} \times \boldsymbol{\Omega})| \end{aligned}$$

or the equivalent basis of Cambon and Jacquin [4] and the corresponding tensors

$$(5.2) \quad \begin{aligned} \xi_{ij}^0 &= e_i^{(1)} e_j^{(2)} - e_j^{(1)} e_i^{(2)} \\ \xi_{ij}^1 &= e_i^{(1)} e_j^{(2)} + e_j^{(1)} e_i^{(2)} \\ \xi_{ij}^2 &= e_i^{(1)} e_j^{(1)} - e_i^{(2)} e_j^{(2)} \\ \xi_{ij}^3 &= e_i^{(1)} e_j^{(1)} + e_i^{(2)} e_j^{(2)} \end{aligned}$$

In Eq. (5.1), $\boldsymbol{\Omega} = 2(0, 0, \Omega)$ is twice the angular velocity. Note that

$$(5.3) \quad \xi_{ij}^3 = P_{ij}(\mathbf{k}) = \delta_{ij} - k_i k_j k^{-2}$$

The linear response function for rotating turbulence is the solution of

$$(5.4) \quad \dot{G}_{ij} + P_{ip} \Omega_{pq} G_{qj} = \delta(t-s) \delta_{ij}$$

which can be written

$$(5.5) \quad \begin{aligned} G_{ij}(\mathbf{k}, t, s) &= \{\cos(2\Omega k_z(t-s)) P_{ij}(\mathbf{k}) \\ &+ \sin(2\Omega k_z(t-s)/k) \xi_{ij}^0(\mathbf{k})\} H(t-s) \end{aligned}$$

where H is the unit step function. At this level of approximation, the *fluctuation-dissipation* theorem,

$$(5.6) \quad Q_{ij}(\mathbf{k}, \tau) = G_{im}(\mathbf{k}, \tau) Q_{mj}(\mathbf{k}) + G_{jm}(\mathbf{k}, -\tau) Q_{mi}(\mathbf{k})$$

applies, where $\tau = t - s$ is time difference.

It must be expected [4] that the single-time correlation function is a sum over all of the ξ tensors of Eq. (5.2)

$$(5.7) \quad Q_{ij}(\mathbf{k}) = \sum Q^p(\mathbf{k}) \xi_{ij}^p(\mathbf{k})$$

But substituting Eq. (5.7) in Eq. (5.6), we find that the symmetry

$$Q_{ij}(\mathbf{k}, \tau) = Q_{ji}(\mathbf{k}, -\tau)$$

leads instead to the simpler expression

$$(5.8) \quad Q_{ij}(\mathbf{k}, \tau) = \sin(2\Omega k_z \tau) Q(\mathbf{k}) \xi_{ij}^0(\mathbf{k}) + \cos(2\Omega k_z \tau) Q(\mathbf{k}) \xi_{ij}^3(\mathbf{k})$$

in which the single-time correlation function is tensorially isotropic. For the purpose of computing the flux integral, we will assume isotropy in \mathbf{k} as well. Although this description of the correlation function is certainly not realistic, the only use we make of it is substitution in the energy flux balance; an isotropic form is adequate for this purpose. In order to evaluate the correlation function, it is necessary to solve closure equations; a complete discussion is given by Cambon and Jacquin [4], although the analytical solution remains unknown.

The energy flux integral, representing the net energy flux across the wavenumber k' is

$$\begin{aligned}
\varepsilon = & \left\{ \int_{k \leq k'} d\mathbf{k} \int_{p, q \geq k'} d\mathbf{p} d\mathbf{q} - \int_{k \geq k'} d\mathbf{k} \int_{p, q \leq k'} d\mathbf{p} d\mathbf{q} \right\} \delta(\mathbf{k} - \mathbf{p} - \mathbf{q}) \int_0^\infty d\tau \times \\
& \{ P_{im'n}(\mathbf{k}) P_{m'rs}(\mathbf{p}) Q_{ir}(\mathbf{k}, \tau) Q_{ns}(\mathbf{q}, \tau) G_{mm'}(\mathbf{p}, \tau) \\
& + P_{imn}(\mathbf{k}) P_{mrs}(\mathbf{q}) Q_{ir}(\mathbf{k}, \tau) Q_{ns}(\mathbf{p}, \tau) G_{mm'}(\mathbf{q}, \tau) \\
& - P_{imn}(\mathbf{k}) P_{i'rs}(\mathbf{k}) Q_{mr}(\mathbf{p}, \tau) Q_{ns}(\mathbf{q}, \tau) G_{ii'}(\mathbf{k}, \tau) \}
\end{aligned}
\tag{5.9}$$

for three-wave interactions, where

$$P_{imn}(\mathbf{k}) = k_m P_{in}(\mathbf{k}) + k_n P_{im}(\mathbf{k}) \tag{5.10}$$

Locality of the inertial range means that this integral is finite when the single-time energy spectrum scaling as $Q(k) \sim k^{-4}$ is substituted. In the absence of resonances, the convergence of the flux integral in this case is well-known: [7] it is a special case of the convergence of the integral in Eq. (5.9) when $Q(k) \sim k^{-\beta-2}$ and $1 < \beta < 3$. However, the resonance conditions require that this condition be checked independently.

5.1. Rotating turbulence. The proposed spectrum falls off rapidly enough at small scales that the only possible divergence occurs when $\mathbf{q} \rightarrow 0$ and $\mathbf{p} \rightarrow \mathbf{k}$. The first question is whether the cancellation of singularities in the absence of resonance conditions survives if the more complex tensor forms of Eq. (5.8) are substituted. The verification is elementary: in this limit, the singularities in Eq. (5.9) are multiplied by

$$P_{imn}(\mathbf{k}) P_{mrs}(\mathbf{k}) \xi_{ir}^{(i)}(\mathbf{k}) \xi_{ns}^{(j)}(0) = k_n k_s P_{ir}(\mathbf{k}) \xi_{ir}^{(i)} \xi_{ns}^{(j)}(0) \tag{5.11}$$

and

$$P_{imn}(\mathbf{k}) P_{irs}(\mathbf{k}) \xi_{mr}^{(i)}(\mathbf{k}) \xi_{ns}^{(j)}(0) = k_n k_s P_{im}(\mathbf{k}) \xi_{im}^{(i)} \xi_{ns}^{(j)}(0) \tag{5.12}$$

In Eqs. (5.11) and (5.12), i and j can each equal 0 or 3. When $j = 0$, both terms vanish, and when $j = 3$, both are proportional to k^2 . Consequently, the terms of order q^0 cancel. Then the integrand in Eq. (5.9) is of order \mathbf{q} . But terms of this order vanish on integration by symmetry. Accordingly, as in the case of isotropic turbulence, the integrals behave like

$$\int q^{2-\beta} dq \tag{5.13}$$

near $\mathbf{q} = 0$ in the absence of resonance conditions. This result depends on the odd parity of the integrand under the substitution $\mathbf{q} \rightarrow -\mathbf{q}$; this parity is preserved by the resonance conditions which are homogeneous of degree zero.

The resonance conditions imply that \mathbf{q} does not approach zero in an arbitrary fashion. Instead, the following cases occur as $\mathbf{q} \rightarrow 0$ and $\mathbf{p} \rightarrow \mathbf{k}$:

$$\begin{aligned}
\omega(\mathbf{k}) + \omega(\mathbf{p}) &= 2\mathbf{k} \cdot \boldsymbol{\omega}(\mathbf{q}) = \pm 2\omega(\mathbf{k}) \\
\omega(\mathbf{k}) - \omega(\mathbf{p}) &= 0, \omega(\mathbf{q}) = 0
\end{aligned}
\tag{5.14}$$

In the first case, \mathbf{q} varies on a cone $q_z = \lambda q$, and in the second case, \mathbf{q} is on a surface tangent to the plane $q_z = 0$. In both cases, the element of area on the surface can be taken proportional to $q dq$. The asymptotics indicated in Eq. (5.13) are replaced by

$$\varepsilon \sim \int q^{1-\beta} dq \tag{5.15}$$

and the flux integral is logarithmically divergent when $\beta = 2$.

It is also necessary to verify that there is no additional divergence contributed by the double curve Eq. (3.10), but this conclusion is immediate because the double curve does not meet the singularity at $\mathbf{q} = 0$.

Thus, the flux integral is logarithmically divergent for the k^{-2} spectrum. It is known [12] that in this case, logarithmic corrections to the spectrum are necessary: instead of Eq. (1.9), the corrected scaling Eq. (1.10) applies.

5.2. Waves in stratified turbulence. It was suggested earlier that the theory of waves in stratified turbulence decoupled from geostrophic motion may not occur in the actual physical system if four-wave interactions can generate geostrophic modes from wave modes. Accordingly, a model problem will be investigated in which the nonlinearity of rotating turbulence is retained, but the dispersion relation is replaced by the dispersion relation of stratified turbulence.

The conditions in Eq. (5.14) apply to this model problem. In the first case, \mathbf{q} approaches zero along a cone $q^\perp = \lambda q$, but in the second case, \mathbf{q} must approach zero tangent to the line $q_x = q_y = 0$. Approach to $\mathbf{q} = 0$ along a line would be extremely singular, but the tangency forces the ratio q^\perp/q to be nonzero, contrary to the assumption that $q^\perp/q = 0$. Thus, the flux integral is only logarithmically divergent, as in the case of rotation. The log-corrected spectrum Eq. (1.10) will apply.

Analogous arguments apply to the coupled problem. However, we stress that a complete treatment of these problems requires an account of the effect of coupling between waves and geostrophic modes.

6. The role of higher order resonances. It was noted earlier that the energy spectrum

$$(6.1) \quad E(k) = C\sqrt{f\varepsilon}k^{-2}$$

where the frequency f is 2Ω for rotation and N for stratification has already been proposed for rotating turbulence, [10] and for stratified turbulence [11]. In the coupled problem, Eq. (5.5) has also been proposed with a suitable function $f = f(\Omega, N)$ [19]. It was noted earlier that in all cases, the heuristic argument is similar to Orszag's [7]: if the time scale is homogeneous of degree zero, simple closure arguments assuming a constant energy flux imply that the energy spectrum scales as k^{-2} .

Weak turbulence theory suggests that such arguments are incomplete. In the isotropic problem of Langmuir turbulence, $\omega \sim k^0$, but the dispersion relation does not permit resonant three-wave interactions [3]. In the absence of three-wave resonances, four-wave processes must be considered. This change modifies the inertial range energy balance. The flux integral Eq. (5.9) for three-wave interactions can be written for scaling purposes as

$$(6.2) \quad \varepsilon \sim \int (d\mathbf{k})^2 P(k) \Theta P(k) Q(k)^2$$

where $P \sim k$ represents the nonlinear coupling, Θ is the time scale, and the number of wavevector integrals is represented symbolically. For four-wave interactions, instead

$$(6.3) \quad \varepsilon \sim \int (d\mathbf{k})^3 P(k) (\Theta P(k))^3 Q(k)^3$$

If $\Theta \sim k^0$, then Eq. (6.2) predicts $E(k) \sim k^{-2}$ but Eq. (6.3) predicts $E(k) \sim k^{-7/3}$. More generally, the spectral scalings of Eqs. (1.6) and (1.7) follow from the flux integrals Eqs. (6.2) and (6.3).

Yakhot [20] has proposed that four-wave resonances play a crucial role in rotating turbulence, especially in the small aspect-ratio systems investigated by Smith *et al.* [21]. It is possible to assess the role of four-wave interactions heuristically from the scaling forms of the energy balance Eqs. (6.2) and (6.3). Suppose

that the modal amplitudes are determined by three-wave interactions as proposed above. These amplitudes can be substituted in the four-wave expression, although different sets of modes resonate in each case. The resulting four-wave correction to the energy flux is of order $k^2\varepsilon/\Omega^3$. Since $\varepsilon^{1/3}k^{2/3}/\Omega \ll 1$ is the condition for weak turbulence theory to apply, the corrections to energy flux due to four-wave interactions would seem to be small.

But this discussion ignores the constants in front of the two flux integrals. In cases in which three-wave interactions are depleted, say by a combination of small aspect-ratio and a very large horizontal viscosity, the four-wave contribution could dominate the three-wave contribution.

In stratified turbulence, the resonance condition again permits three-wave resonances, and it has been shown that the double resonances are irrelevant to energy transfer. But in this problem, the importance of interactions with geostrophic modes [5] and the comparatively secondary role of pure wave turbulence, suggests that a more detailed analysis is required. The turbulence of nearly horizontal wave modes has recently been analyzed by Caillol and Zeitlin [22] on the basis of weak turbulence theory.

The same considerations apply to the coupled problem. But in this case, the dispersion relation Eq. (3.18) degenerates when $N = 2\Omega$ to the isotropic form $\omega = N$ which excludes three-wave resonances. Wave turbulence in this case must be analyzed in terms of four-wave interactions, but since any four wavevectors can form a resonant quartet, it is not possible to derive a spectral scaling from the flux integral in the weak turbulence approximation without making further assumptions. In view of the greater dynamic importance of the interactions of waves with geostrophic modes, the role of four-wave interactions in rotating stratified turbulence remains uncertain.

Appendix I. If $p_x = 0$, then either $q_x = 0$ or $q_z = 0$. If $p_x = q_x = 0$, then if $p_y \neq 0$, dividing the last condition in Eq. (3.4) by the second, $p_z/p_y = q_z/q_y$. This reduces to the case of proportionality of the wavevectors, which we have seen does not yield a double resonance. If instead, $p_y = 0$, then the third of Eq. (3.4) forces also $q_y = 0$, and we have two vertical vectors. This case has also been shown not to lead to double resonances.

Suppose next that $p_x = 0$ and $q_z = 0$. Then either $p_y = 0$ or $p_z = 0$. In the first case, $p_x = p_y = 0$ implies $q_x = q_y = 0$, which does not lead to a double resonance. In the second case, $p_z = q_z = 0$, and this case has already been considered.

Other cases correspond to interchanging x and y . The only double resonance is therefore the double line Eq. (3.10).

Appendix II. If $p_x = 0$ and $q_x = 0$, then $p_y = \pm p^\perp$ and $q_y = \pm q^\perp$. Multiply the second relation of Eq. (3.12) by p^\perp/p^\perp on the left side and by q^\perp/q^\perp on the right side, then use the third relation to conclude that $p_z/p_y = q_z/q_y$. This is the case of collinearity, which we know cannot satisfy the resonance condition.

If $p_x = 0$ and $q_z = 0$ then either $p_z = 0$ or $p_y = 0$. In the first case, $p_z = q_z = 0$ which has already been analyzed. In the second case, we have $\mathbf{p} = (0, 0, p_z)$, $\mathbf{q} = (q_x, q_y, 0)$ and then \mathbf{p}, \mathbf{q} , and \mathbf{k} cannot resonate.

The remaining cases again correspond to interchanging x and y . The point singularity found in the main text is therefore the only multiple resonance.

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